## 4.3 Review: Trigonometric Functions of Any Angle

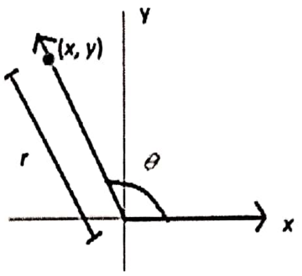
**Definitions of Trigonometric Functions of Any Angle**

Let $\theta$ be an angle in standard position with (x, y) a point on the terminal side of $\theta$ and $r = \sqrt{x^2 + y^2} \neq 0$.

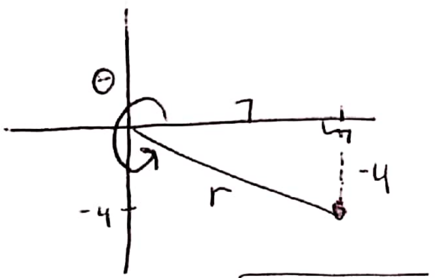
$\sin\theta = \frac{y}{r}$ $\qquad \cos\theta = \frac{x}{r}$

$\tan\theta = \frac{y}{x}, \quad x \neq 0$ $\qquad \cot\theta = \frac{x}{y}, \quad y \neq 0$

$\sec\theta = \frac{r}{x}, \quad x \neq 0$ $\qquad \csc\theta = \frac{r}{y}, \quad y \neq 0$




**Example 1: Let (7, -4) be a point on the terminal side of $\theta$. Find the sine, cosine, and tangent of $\theta$.**




$r = \sqrt{(7)^2 + (-4)^2}$

$r = \sqrt{65}$

$\sin\theta = \frac{-4}{\sqrt{65}}$

$\cos\theta = \frac{7}{\sqrt{65}}$

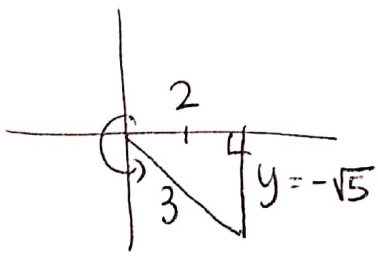
$\tan\theta = -\frac{4}{7}$

**Example 2: Given $\cos\theta = \frac{2}{3}$ and $\tan\theta < 0$, find $\csc\theta$.**

$\left.\begin{array}{l} \cos\theta \text{ pos} \\ \tan\theta \text{ neg.} \end{array}\right]$ QIV

$\csc\theta = \frac{1}{\sin\theta} = \frac{r}{y} = \frac{3}{-\sqrt{5}}$

$\csc\theta = -\frac{3}{\sqrt{5}}$




$2^2 + y^2 = 3^2$

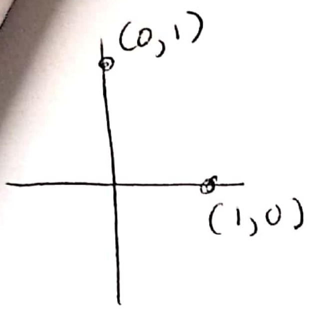
$4 + y^2 = 9$

$y^2 = 5$

$y = \sqrt{5}$ in QIV

$-\sqrt{5}$

**Example 3: Evaluate the cosecant and cotangent functions at $0$ and $\frac{\pi}{2}$.**




$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{y}$

$\csc(0) = \frac{1}{0} = \text{undef.}$

$\csc\left(\frac{\pi}{2}\right) = \frac{1}{1} = 1$

$\cot\theta = \frac{x}{y}$

$\cot(0) = \frac{1}{0}$ undef.

$\cot\left(\frac{\pi}{2}\right) = \frac{0}{1} = 0$

**Definition of Reference Angles**

Let $\theta$ be an angle in standard position. Its **reference angle** is the acute angle $\theta'$ formed by the terminal side of $\theta$ and the horizontal axis.

**In the space below, sketch a graph of the reference angle $\theta$ in Quadrants II, III, and IV.**

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**Example 4: Find the reference angle $\theta'$. Sketch a graph of each reference angle.**

**a.** $\theta = \frac{7\pi}{9}$ **b.** $\theta = 1.7$ **c.** $\theta = 144°$

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**Evaluating Trigonometric Functions of Any Angle**

To find the value of a trigonometric function of any angle $\theta$:

1. Determine the function value of the associated reference angle $\theta'$.
2. Depending on the quadrant in which $\theta$ lies, affix the appropriate sign to the function value.

**Trigonometric Values of Common Angles**

| $\theta$ (degrees) | 0° | 30° | 45° | 60° | 90° | 180° | 270° |
|---|---|---|---|---|---|---|---|
| $\theta$ (radians) | $0$ | $\pi/6$ | $\pi/4$ | $\pi/3$ | $\pi/2$ | $\pi$ | $3\pi/2$ |
| $\sin\theta$ $y$ | $0$ | $1/2$ | $\sqrt{2}/2$ | $\sqrt{3}/2$ | $1$ | $0$ | $-1$ |
| $\cos\theta$ $x$ | $0$ | $\sqrt{3}/2$ | $\sqrt{2}/2$ | $1/2$ | $0$ | $-1$ | $0$ |
| $\tan\theta$ | $0$ | $\sqrt{3}/3$ or $\frac{1}{\sqrt{3}}$ | $1$ | $\sqrt{3}$ | undef | $0$ | undef |

**Example 5: Evaluate each trigonometric function.**

**a.** $\sin\frac{5\pi}{3}$

QIV

$-\frac{\sqrt{3}}{2}$

**b.** $\cos\left(-60^\circ\right)$

QIV

$\frac{1}{2}$

**c.** $\tan\frac{17\pi}{6}$

$\frac{12\pi}{6} + \frac{5\pi}{6}$

QII

$-\frac{1}{\sqrt{3}} = \frac{-\sqrt{3}}{3}$

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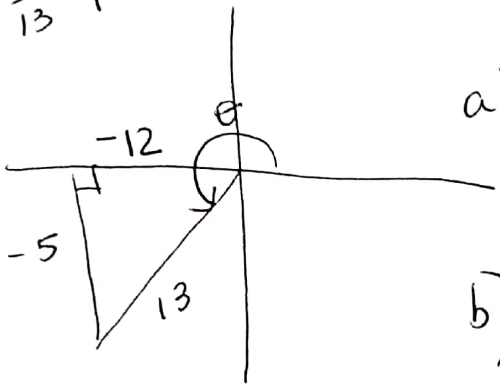
**Example 6: Let $\theta$ be an angle in Quadrant III such that $\sin\theta = -\frac{5}{13}$. Find (a) $\sec\theta$ and (b) $\tan\theta$ ~~by using trigonometric identities.~~** Draw Picture!

$\sin\theta = \frac{-5}{13}$ $\frac{y}{r}$




a) $\sec\theta = \frac{r}{x} = -\frac{13}{12}$

b) $\tan\theta = \frac{y}{x} = \frac{5}{12}$

$x^2 + (-5)^2 = 13^2$

$X = 12$ in QIII $-12$

**Example 7: Use a calculator to evaluate each trigonometric function.**

**a.** $\cot 375°$ **b.** $\sin(-4.1)$ **c.** $\sec\frac{3\pi}{8}$

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ors Pre-Calculus Section 4.4 Practice    Name:____________________________________    Period:_______

ection 4.4 Graphing Sine and Cosine Functions

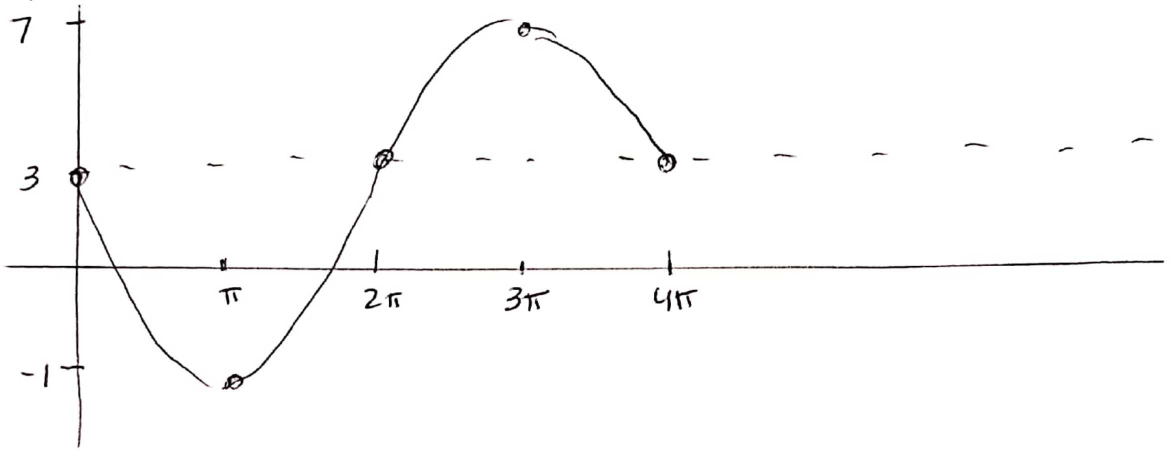
**SHOW ALL WORK! (Critical points: min, max, and intercepts of the midline or x-axis)**

1.) Graph **one full period** of the following function:

$$y = 3 - 4\sin\frac{x}{2}$$

Amplitude 4   Period $4\pi$   Phase Shift/Left Endpoint 0   Right Endpoint $4\pi$   Vertical Shift 3

Graph:



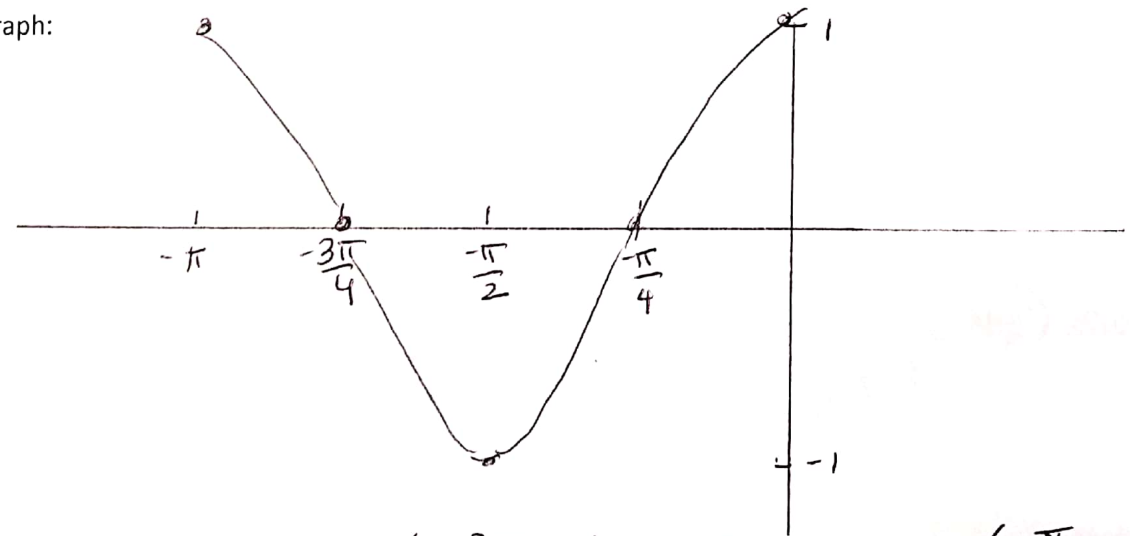

Critical Points $(0,3)$ $(\pi,-1)$ $(2\pi,3)$ $(3\pi,7)$ $(4\pi,3)$

2.) Graph **one full period** of the following function:

$$y = \cos(2x + 2\pi)$$

Amplitude 1   Period $\pi$   Phase Shift/Left Endpoint $-\pi$   Right Endpoint 0   Vertical Shift 0

Graph:




Critical Points $(-\pi,1)$ $(-\frac{3\pi}{4},0)$ $(-\pi/2,-1)$ $(-\pi/4,0)$ $(0,1)$

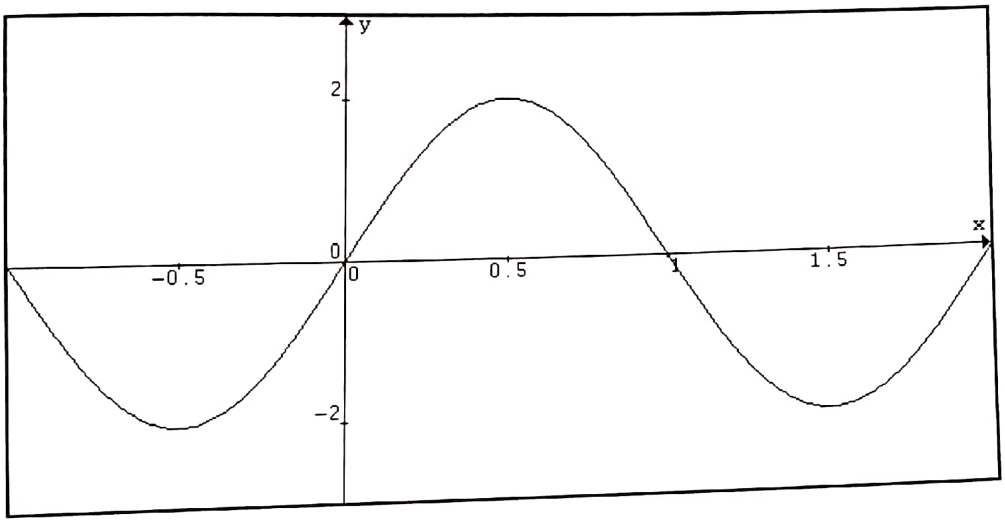
e the function of the sine graph described below:

The function $f(x)$ has a vertical shift of -9, completes one full period in $2\pi$ radians, has a phase shift of $\pi$, and has an amplitude of 3.

(Right $\pi$)

$$f(x) = 3\sin(x - \pi) - 9$$

4.) Answer the following questions about the graph below:




What is the:

Amplitude $2$  Period $2$  Phase Shift $0 / -\frac{1}{2}$ - left $\frac{1}{2}$ Reflected cosine  Midline/ Vertical Shift $0$

Equation $y = 2\sin(\pi x)$ or

$\pi(x + 1/2)$

$b = \pi$

$$y = -2\cos\left(\pi x + \frac{\pi}{2}\right) \text{ or}$$

$\pi(x - 1/2)$

$$y = 2\cos\left(\pi x - \frac{\pi}{2}\right)$$ (P/s right 1/2)